The Physics of Racing, Part 3: Basic Calculations

Brian Beckman

physicist and member of
No Bucks Racing Club

P.O. Box 662
Burbank, CA 91503

In the last two articles, we plunged right into some relatively complex issues, namely weight transfer and tire adhesion. This month, we regroup and review some of the basic units and dimensions needed to do dynamical calculations. Eventually, we can work up to equations sufficient for a full-blown computer simulation of car dynamics. The equations can then be 'doctored' so that the computer simulation will run fast enough to be the core of an autocross computer game. Eventually, we might direct this series of articles to show how to build such a game in a typical microcomputer programming language such as C or BASIC, or perhaps even my personal favorite, LISP. All of this is in keeping with the spirit of the series, the Physics of Racing, because so much of physics today involves computing. Software design and programming are essential skills of the modern physicist, so much so that many of us become involved in computing full time.

Physics is the science of measurement. Perhaps you have heard of highly abstract branches of physics such as quantum mechanics and relativity, in which exotic mathematics is in the forefront. But when theories are taken to the laboratory (or the race course) for testing, all the mathematics must boil down to quantities that can be measured. In racing, the fundamental quantities are distance, time, and mass. This month, we will review basic equations that will enable you to do quick calculations in your head while cooling off between runs. It is very valuable to develop a skill for estimating quantities quickly, and I will show you how.

Equations that don't involve mass are called *kinematic*. The first kinematic equation relates speed, time, and distance. If a car is moving at a constant speed or velocity, v, then the distance d it travels in time t is

$$d = vt$$

or velocity times time. This equation really expresses nothing more than the definition of velocity.

If we are to do mental calculations, the first hurdle we must jump comes from the fact that we usually measure speed in miles per hour (mph), but distance in feet and time in seconds. So, we must modify our equation with a conversion factor, like this

$$d\,(\text{feet}) = v\frac{\text{miles}}{\text{hour}}\,t\,(\text{seconds})\frac{5280\text{ feet/mile}}{3600\text{ seconds/hour}}$$

If you "cancel out" the units parts of this equation, you will see that you get feet on both the left and right hand sides, as is appropriate, since equality is required of any equation. The conversion factor is 5280/3600, which happens to equal 22/15. Let's do a few quick examples. How far does a car go in one second (remember, say, "one-one-thousand, two-one-thousand," *etc.* to yourself to count off seconds)? At fifteen mph, we can see that we go

$$d = 15\text{ mph times 1 sec times } 22/15 = 22\text{ feet}$$

or about 1 and a half car lengths for a 14 and 2/3 foot car like a late-model Corvette. So, at 30 mph, a second is three car lengths and at 60 mph it is six. If you lose an autocross by 1 second (and you'll be pretty good if you can do that with all the good drivers in our region), you're losing by somewhere between 3 and 6 car lengths! This is because the average speed in an autocross is between 30 and 60 mph.

Everytime you plow a little or get a little sideways, just visualize your competition overtaking you by a car length or so. One of the reasons autocross is such a difficult sport, but also such a pure sport, from the driver's standpoint, is that you can't make up this time. If you blow a corner in a road race, you may have a few laps in which to make it up. But to win an autocross against good competition, you must drive nearly perfectly. The driver who makes the fewest mistakes usually wins!

The next kinematic equation involves acceleration. It so happens that the distance covered by a car at constant acceleration from a standing start is given by

$$d = \frac{1}{2}at^2$$

or 1/2 times the acceleration times the time, squared. What conversions will help us do mental calculations with this equation? Usually, we like to measure acceleration in Gs. One G happens to be 32.1 feet per second squared. Fortunately, we don't have to deal with miles and hours here, so our equation becomes,

$$d\,(\text{feet}) = 16a\,(G\text{s})\,t\,(\text{seconds})^2$$

roughly. So, a car accelerating from a standing start at $\frac{1}{2}G$, which is a typical number for a good, stock sports car, will go 8 feet in 1 second. Not very far! However, this picks up rapidly. In two seconds, the car will go 32 feet, or over two car lengths.

Just to prove to you that this isn't crazy, let's answer the question "How long will it take a car accelerating at $\frac{1}{2}G$ to do the quarter mile?" We invert the equation above (recall your high school algebra), to get

$$t = \sqrt{d\,(\text{feet})\ 16a\,(G\text{s})}$$

and we plug in the numbers: the quarter mile equals 1320 feet, $a = \frac{1}{2}G$, and we get $t = \sqrt{1320/8} = \sqrt{165}$ which is about 13 seconds. Not too unreasonable! A real car will not be able to keep up full $\frac{1}{2}G$ acceleration for a quarter mile due to air resistance and reduced torque in the higher gears. This explains why real (stock) sports cars do the quarter mile in 14 or 15 seconds.

The more interesting result is the fact that it takes a full second to go the first 8 feet. So, we can see that the launch is critical in an autocross. With excessive wheelspin, which robs you of acceleration, you can lose a whole second right at the start. Just visualize your competition pulling 8 feet ahead instantly, and that margin grows because they are 'hooked up' better.

For doing these mental calculations, it is helpful to memorize a few squares. 8 squared is 64, 10 squared is 100, 11 squared is 121, 12 squared is 144, 13 squared is 169, and so on. You can then estimate square roots in your head with acceptable precision.

Finally, let's examine how engine torque becomes force at the drive wheels and finally acceleration. For this examination, we will need to know the mass of the car. Any equation in physics that involves mass is called *dynamic*, as opposed to kinematic. Let's say we have a Corvette that weighs 3200 pounds and produces 330 foot-pounds of torque at the crankshaft. The Corvette's automatic transmission has a first gear ratio of 3.06 (the auto is the trick set up for vettes—just ask Roger Johnson or Mark Thornton). A transmission is nothing but a set of circular, rotating levers, and the gear ratio is the leverage, multiplying the torque of the engine. So, at the output of the transmission, we have

$$3.06 \times 330 = 1010 \, \text{foot-pounds}$$

of torque. The differential is a further lever-multiplier, in the case of the Corvette by a factor of 3.07, yielding 3100 foot pounds at the center of the rear wheels (this is a lot of torque!). The distance from the center of the wheel to the ground is about 13 inches, or 1.08 feet, so the maximum force that the engine can put to the ground in a rearward direction (causing the ground to push back forward—remember part 1 of this series!) in first gear is

$$3100 \, \text{foot-pounds}/1.08 \, \text{feet} = 2870 \, \text{pounds}$$

Now, at rest, the car has about 50/50 weight distribution, so there is about 1600 pounds of load on the rear tires. You will remember from last month's article on tire adhesion that the tires cannot respond with a forward force much greater than the weight that is on them, so they simply will spin if you stomp on the throttle, asking them to give you 2870 pounds of force.

We can now see why it is important to squeeeeeeeze the throttle gently when launching. In the very first instant of a launch, your goal as a driver is to get the engine up to where it is pushing on the tire contact patch at about 1600 pounds. The tires will squeal or hiss just a little when you get this right. Not so coincidentally, this will give you a forward force of about 1600 pounds, for an $F = ma$ (part 1) acceleration of about $\frac{1}{2}G$, or half the weight of the car. The main reason a car will accelerate with only $\frac{1}{2}G$ to start with is that half of the weight is on the front wheels and is unavailable to increase the stiction of the rear, driving tires. Immediately, however, there will be some weight transfer to the rear. Remembering part 1 of this series again, you can estimate that about 320 pounds will be transferred to the rear immediately. You can now ask the tires to give you a little more, and you can gently push on the throttle. Within a second or so, you can be at full throttle, putting all that torque to work for a beautiful hole shot!

In a rear drive car, weight transfer acts to make the driving wheels capable of withstanding greater forward loads. In a front drive car, weight transfer works against acceleration, so you have to be even more gentle on the throttle if you have a lot of power. An all-wheel drive car puts all the wheels to work delivering force to the ground and is theoretically the best.

Technical people call this style of calculating "back of the envelope," which is a somewhat picturesque reference to the habit we have of writing equations and numbers on any piece of paper that happens to be handy. You do it without calculators or slide rules or abacuses. You do it in the garage or the pits. It is not exactly precise, but gives you a rough idea, say within 10 or 20 percent, of the forces and accelerations at work. And now you know how to do back-of-the-envelope calculations, too.